CONTRIBUTION TO THE THEORY OF HEAT EXCHANGE IN A LAYER WITH PERIODIC CHANGE OF THE DIRECTION OF MOTION OF THE LIQUID

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We formulate the problem of heat exchange in a layer of dispersed matter under periodic variation of the direction of motion of the liquid through the medium, with allowance for the thermal resistance of the material of the particles in the layer, and under conditions when the process proceeds without a gradient.

Although heat exchange in a layer of disperse material has been dealt with for several decades, many problems remain unresolved to this day. The heat exchange in a layer has been considered in greatest detail in the basic monographs [1-5], and also in [6]. In no known work, however, has the problem of heat exchange in a layer of disperse material been formulated for periodic variation of the direction of motion of the liquid through the layer. We consider in this connection a heat-exchange process that follows the scheme shown in Fig. 1. During the first period of time, $0 \le \tau \le \tau_1$, the motion of the liquid through the particle layer is in the direction of the solid arrow, and the temperature of the liquid ahead of the layer is t_0. During the next time interval $\tau_1 \le \tau < \tau_1 + \tau_2$ the liquid flows in the opposite direction, shown by the dashed arrow, entering the layer at a temperature t_h; in the general case $t_0 \ne t_h$ and $\tau_1 \ne \tau_2$. In the next time interval $\tau_1 + \tau_2 \le \tau < 2\tau_1 + \tau_2$ the liquid again moves in the direction of the solid arrow, the initial temperature of the liquid is t_0, etc. Thus, the initial temperature of the liquid prior to entering the layer is a periodic function of two values of the coordinate x ($x = 0$ and $x = h$), which in turn are functions of the time τ. Such a variation of the direction of motion of liquids (gases) with different initial temperatures t_0 and t_h is shown graphically in Fig. 2.

Such a process can occur in regenerative heat exchange [7], in the presence of internal heat sources, or in coupled heat and mass transfer in drying [8], adsorption [9], diffusion extraction of matter [10], and other technological processes.

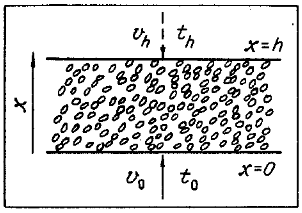

Fig. 1

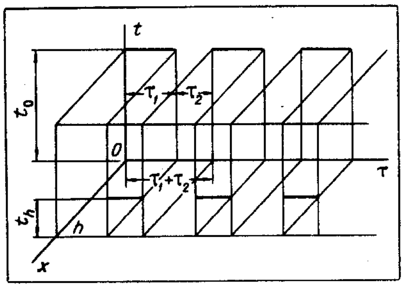

Fig. 2

Fig. 1. Theoretical scheme of heat exchange in a layer in the case of reversal.

Fig. 2. Graphical representation of the variation of the initial temperature of the liquid under reversal.

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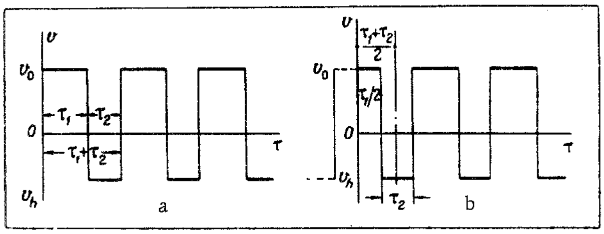

Fig. 3. Variation of the initial velocity of the liquid. a) Start of cycle coincides with the origin; b) illustrating the determination of the Fourier series coefficients.

Such a process is usually calculated in the following manner. In the case of noncoupled transport the final temperature of the packing or the concentration of the material, and in the case of coupled heat and mass transfer the final temperature of the packing and the final value of the concentration of the material, attained at the end of the first period of the process after a time τ_2, are taken to be the initial distributions of these quantities for the second period of time τ_2. The analysis is confined most frequently to these periods, since otherwise the problem becomes so complicated that no means of solution is known. Such an approach to the calculation of the temperature of the packing at any instant of time for arbitrary repeating periods will give results relatively easily only if the liquids move in one direction, as shown in [11-13].

We formulate below the problem for the case of heat exchange in the process described and illustrated in Figs. 1 and 2. The system of equations for coupled heat and mass transfer is given in most complete form in [14]. However, since small changes are introduced into the main system of equations in our case, and the periodic change of direction of the liquid motion is reflected in the boundary conditions, we consider only the simplest case of uncoupled heat and mass transfer in the layer.

Making the usual assumptions for the process in question and taking into account the thermal resistance of the material of the particles of the layer, the system of heat-exchange equations takes the form

$$\frac{\partial\vartheta(r,x,\tau)}{\partial\tau} = a\left[\frac{\partial^2\vartheta(r,x,\tau)}{\partial r^2} + \frac{\Gamma}{2}\,\frac{\partial\vartheta(r,x,\tau)}{\partial r}\right], \tag{1}$$

$$v(\tau)\,\frac{\partial t(x,\tau)}{\partial x} + \frac{\partial t(x,\tau)}{\partial\tau} + \frac{\alpha F}{c_g m}\,[t(x,\tau) - \vartheta(R,x,\tau)] = 0, \tag{2}$$

$$-\lambda\left[\frac{\partial\vartheta(r,x,\tau)}{\partial r}\right]_{r=R} = \alpha\,[\vartheta(R,x,\tau) - t(x,\tau)], \tag{3}$$

$$\vartheta(r,x,0) = \vartheta_0, \quad \left.\frac{\partial\vartheta(r,x,\tau)}{\partial r}\right|_{r=R} = 0. \tag{4}$$

Here $\Gamma = 2\nu + 1$ ($\nu = 1/2$ for a sphere, $\nu = 0$ for a cylinder, and $\nu = -1/2$ for a plate). The system (1)-(4) should be supplemented by a boundary condition, which can be written for a periodic variation of the direction of liquid motion (reversal) in the form

$$t[x(\tau),\tau] = t_0\cos\frac{\pi}{2h}\,x(\tau) + t_h\sin\frac{\pi}{2h}\,x(\tau). \tag{5}$$

We denote the ratio of the time of passage of the liquid through the layer in one direction (dashed arrow of Fig. 1) to the time of passage of the liquid in the opposite direction (solid arrow) by $\mu = \tau_2/\tau_1$. Then the periodic function $x(\tau)$ can be represented as follows:

$$x(\tau) = \begin{cases} 0, & \tau_1(n + n\mu) \leqslant \tau < \tau_1[(n+1) + n\mu], \\ h, & \tau_1[(n+1) + n\mu] \leqslant \tau < \tau_1[(n+1)(\mu+1)] \\ & (n = 0, 1, 2, 3, \ldots). \end{cases} \tag{6}$$

It is assumed in (6) that the time intervals during which the liquid moves in a qiven direction are equal. The function $t[x(\tau), \tau]$ described by relations (5) and (6) is a scalar quantity and consequently establishes only the temperature of the liquid prior to entering the layer for two values of the coordinate ($x = 0$ and $x = h$), but does not characterize the direction of liquid motion. Therefore the liquid velocity, which is a vector quantity reflecting the direction of the liquid motion, should also be a periodic function of the time

$$v(\tau) = \begin{cases} +v_0, & \tau_1(n + n\mu) \leqslant \tau < \tau_1[(n+1) + n\mu], \\ -v_h, & \tau_1[(n+1) + n\mu] \leqslant \tau < \tau_1[(n+1)(\mu+1)] \\ & (n = 0, 1, 2, 3, \ldots). \end{cases} \tag{7}$$

Here $+v_0$ is the velocity of the liquid in one direction (solid arrow, Fig. 1), $-v_h$ is the liquid velocity in the opposite direction (dashed arrow, Fig. 1), and in the general case $|v_0| \neq |v_h|$. Quite frequently, cases when $|v_0| = |v_h|$ are encountered in applications; we can then write $v(\tau) = v\psi(\tau)$. It is assumed here that both the velocity of the liquid through the layer and the initial velocities of the liquid are constant quantities, i.e., $v = \text{const}$. The function $v(\tau)$ at a constant value of the velocity on entering the layer is shown in Fig. 3a. The functions $x(\tau)$ and $\psi(\tau)$ can be plotted similarly. The function $\psi(\tau)$ is also periodic,

$$\psi(\tau) = \begin{cases} +1, & \tau_1(n+n\mu) \leqslant \tau < \tau_1[(n+1)+n\mu], \\ -1, & \tau_1[(n+1)+n\mu] \leqslant \tau < \tau_1[(n+1)(\mu+1)] \\ & (n = 0, 1, 2, 3, \ldots). \end{cases} \tag{8}$$

In the case $v = \text{const}$, the system (1)-(5) can be represented in dimensionless form, using the following notation:

$$\rho = \frac{r}{R}; \quad \mathrm{Bi} = \frac{\alpha R}{\lambda}; \quad \xi = \frac{\alpha F}{c_g v m} x; \quad \eta = \frac{a}{R^2}\tau; \tag{9}$$

$$\Theta(\rho, \xi, \eta) = \frac{\vartheta(\rho, \xi, \eta) - \vartheta_0}{t_0 - \vartheta_0}; \quad T(\xi, \eta) = \frac{t(\xi, \eta) - \vartheta_0}{t_0 - \vartheta_0}.$$

We then obtain in place of (1)-(5)

$$\frac{\partial\Theta(\rho, \xi, \eta)}{\partial\eta} = \frac{\partial^2\Theta(\rho, \xi, \eta)}{\partial\rho^2} + \frac{\Gamma}{\rho}\frac{\partial\Theta(\rho, \xi, \eta)}{\partial\rho}, \tag{10}$$

$$k_1\frac{\partial T(\xi, \eta)}{\partial\eta} + \psi(\eta)\frac{\partial T(\xi, \eta)}{\partial\xi} + T(\xi, \eta) - \Theta(1, \xi, \eta) = 0, \tag{11}$$

$$\left.\frac{\partial\Theta(\rho, \xi, \eta)}{\partial\rho}\right|_{\rho=1} = \mathrm{Bi}\,[T(\xi, \eta) - \Theta(1, \xi, \eta)], \tag{12}$$

$$\Theta(\rho, \xi, 0) = 0, \quad \left.\frac{\partial\Theta(\rho, \xi, \eta)}{\partial\rho}\right|_{\rho=0} = 0, \tag{13}$$

$$T[\xi(\eta), \eta] = T_0\cos\frac{\pi}{2\xi^*}\xi(\eta) + T_{\xi^*}\sin\frac{\pi}{2\xi^*}\xi(\eta), \tag{14}$$

where

$$k_1 = \frac{a}{R^2}\cdot\frac{cgm}{\alpha F}, \quad T_0(0, \eta) = \frac{t(0, \eta) - \vartheta_0}{t_0 - \vartheta_0} = 1,$$

$$T_{\xi^*}(\xi^*, \eta) = \frac{t(\xi^*, \eta) - \vartheta_0}{t_0 - \vartheta_0} = \frac{t_h - \vartheta_0}{t_0 - \vartheta_0}.$$

Here

$$\xi^* = \xi|_{x=h}, \quad t(0, \eta) = t_0.$$

The functions $x(\tau)$, $v(\tau)$, and $\psi(\tau)$ can be expanded in a Fourier series. To this end, they should be represented in the form shown in Fig. 3b, but shifted by $\tau_1/2$ relative to the ordinate axis. In such a case they can be expanded only in even functions, and there will consequently be no discontinuity at $\tau = 0$. We expand the functions $x(\tau)$ and $\psi(\tau)$ in a Fourier series for the dimensionless parameters. The coefficients of the series are determined as follows:

$$a_0 = \frac{4}{\mathrm{H}}\int_0^{\mathrm{H}/2}\xi(\eta)d\eta = \frac{4}{\mathrm{H}}\left[\int_0^{\eta_1/2}\xi(0)\,d\eta + \int_{\eta_1/2}^{\mathrm{H}/2}\xi(\xi^*)\,d\eta\right]$$

$$= \frac{2}{1+\mu}[\xi(0) + \mu\xi(\xi^*)] = \frac{2}{1+\mu}\mu\xi^*.$$

Since $\xi(0) = 0$; $\xi(\xi^*) = \xi^*$; $\mathrm{H} = \eta_1 + \eta_2$ (η_1 and η_2 are the dimensionless times corresponding to τ_1 and τ_2, respectively).

Analogously

$$a_m = \frac{4}{H}\left[\int\limits_0^{\eta_1/2} \xi(0)\cos\frac{2\pi m}{H}\eta d\eta + \int\limits_{\eta_1/2}^{H/2} \xi(\xi^*)\cos\frac{2\pi m}{H}\eta d\eta\right]$$

$$= -\frac{2}{\pi m}\xi^*\sin\frac{\pi m}{1+\mu}.$$

We thus have

$$\xi(\eta) = \frac{1}{1+\mu}\mu\xi^* - \sum_{m=1}^{\infty}\frac{2}{\pi m}\xi^*\sin\frac{\pi m}{1+\mu}\cos\frac{2\pi m}{H}\eta. \tag{15}$$

Analogously

$$\psi(\eta) = \frac{1-\mu}{1+\mu} + \sum_{m=1}^{\infty}\frac{4}{\pi m}\sin\frac{\pi m}{1+\mu}\cos\frac{2\pi m}{H}\eta. \tag{16}$$

For the velocity of the liquid we have

$$v(\tau) = \frac{1}{1+\mu}(v_0 - \mu v_h) + \sum_{m=1}^{\infty}\frac{2}{\pi m}(v_0 + v_h)\sin\frac{\pi m}{1+\mu}\cos\frac{2\pi m}{T_{\text{sum}}}\tau, \tag{17}$$

where

$$T_{\text{sum}} = \tau_1 + \tau_2.$$

We note that the temperature and velocity of the liquid prior to entering the layer (t_0, t_h, v_0, v_k), i.e., at $x = 0$ and $x = h$, can also be variable quantities. All the arguments advanced above remain in force, but to be expandable in a Fourier series the function $v(\tau)$ must satisfy the Dirichlet conditions. If we neglect the internal thermal resistance over the volume of the layer particles, then we obtain in the presence of an internal heat source $\varphi(x, \tau)$ (e.g., in the case of drying), in place of (1)-(4)

$$v(\tau)\frac{\partial t(x,\tau)}{\partial x} + \frac{\partial t(x,\tau)}{\partial \tau} + \varphi(x,\tau) = -\frac{\alpha F}{c_g m}[t(x,\tau) - \vartheta(x,\tau)], \tag{18}$$

$$\frac{\partial\vartheta(x,\tau)}{\partial\tau} = \frac{\alpha F}{c(1-m)}[t(x,\tau) - \vartheta(x,\tau)] \tag{19}$$

with initial conditions

$$\vartheta(x,0) = \vartheta_0;\ t(x,0) = \vartheta_0 + (t_0 - \vartheta_0)\exp\left(-\frac{\alpha F}{c_g m v}x\right) \tag{20}$$

and with boundary condition (5). If $|v_0| = |v_h|$, i.e., if $v(\tau) = v\psi(\tau)$, Eqs. (18)-(20) can be reduced to the dimensionless form

$$\psi(z)\frac{\partial T(\xi,z)}{\partial\xi} + k_2\frac{\partial T(\xi,z)}{\partial z} + \varphi(\xi,z) = -[T(\xi,z) - \Theta(\xi,z)], \tag{21}$$

$$\frac{\partial\Theta(\xi,z)}{\partial z} = T(\xi,z) - \Theta(\xi,z), \tag{22}$$

$$\Theta(\xi,0) = 0,\ T(\xi,0) = \exp(-\xi) \tag{23}$$

with boundary condition (14), in which η should be replaced by z, with

$$z = \frac{\alpha F}{c(1-m)}\tau, \text{ and } k_2 = \frac{c_g m}{c(1-m)}.$$

NOTATION

a	thermal diffusivity, m^2/h;
c_g, c	bulk specific heat of the liquid (gas) and material of the particles, $kcal/m^3 \cdot deg$;
F	surface area of particles per unit volume of the layer, m^2/m^3;
h	height of layer, m;
m	porosity, m^2/m^2;
R	characteristic dimension, m;
t	running temperature of the liquid, °C;
v	actual velocity of the liquid through the layer, m/h;
x	running coordinate, m;
α	coefficient of heat exchange between the liquid and the particles of the layer, $kcal/m^2 \cdot h \cdot deg$;
λ	thermal conductivity coefficient, $kcal/m \cdot h \cdot deg$;
τ	time, h;
ϑ	running temperature of the layer particles, °C.

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